# Pulley Mechanism
# for Muscle or Tendon Movements
# along Bones and around Joints

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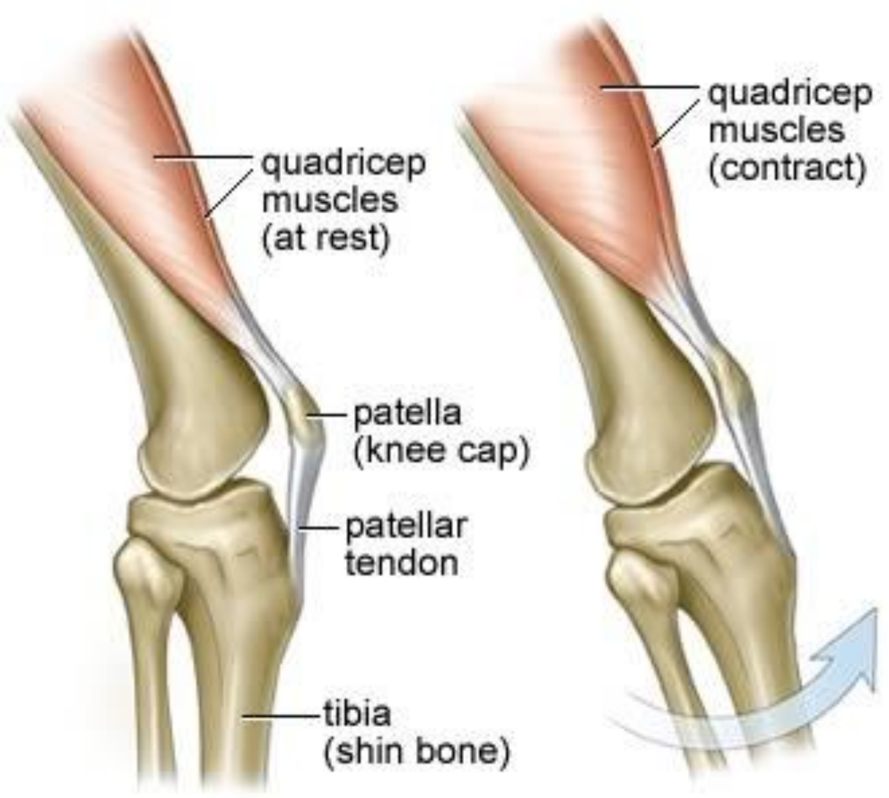
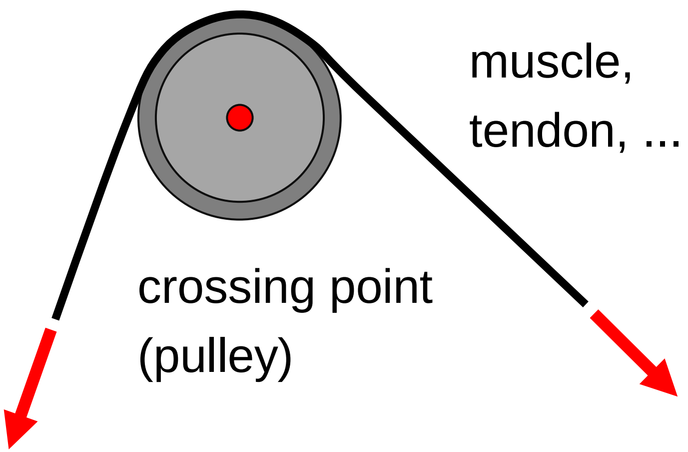
DYNA MORE

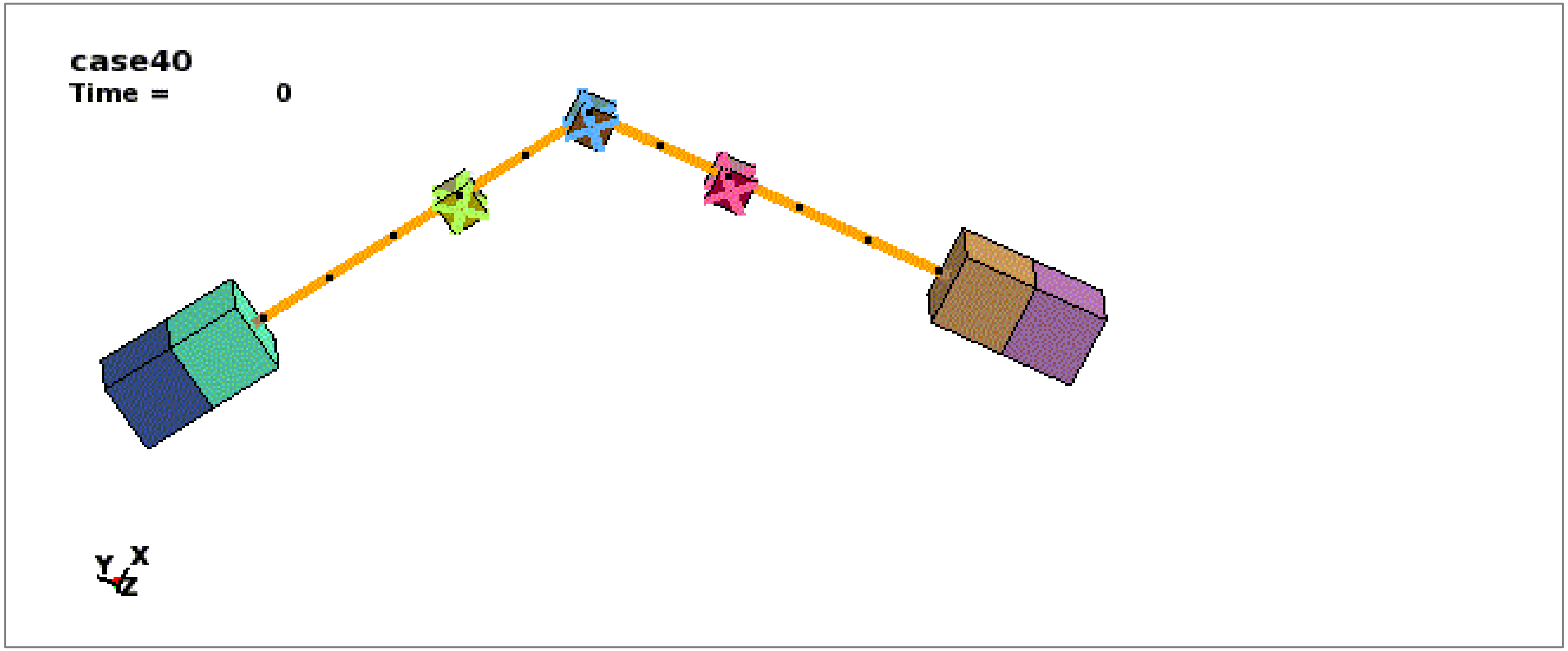
- **Motivation:** FEM model of bent muscles or tendons which are guided along bones and around joints, e.g. ankle, elbow, knee, …
  From an engineering point of view, this is a pulley-like mechanism.



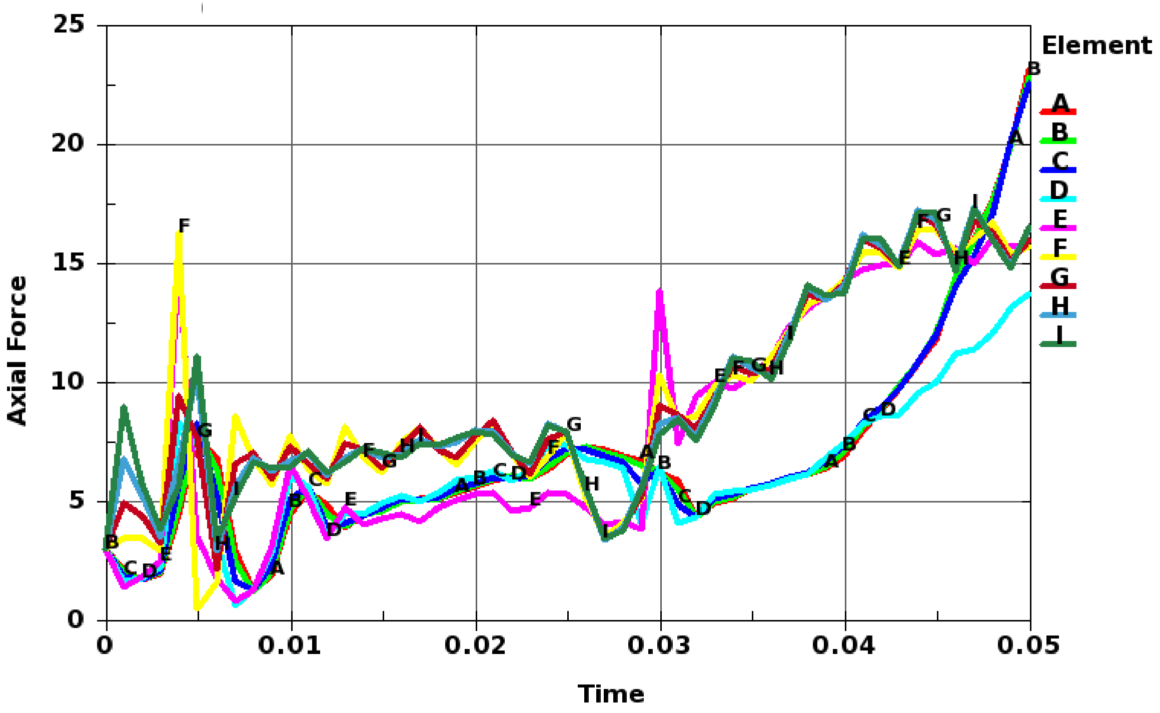

➡ Quantities of interest:
friction, angle, forces, …

* Currently, one could use truss elements with *MAT_MUSCLE and *CONTACT_GUIDED_CABLE:




**Non-smooth results**

- Currently, one could use truss elements with *MAT_MUSCLE and *CONTACT_GUIDED_CABLE:




**Axial forces are not uniform**

- **Idea:** Transfer the slipring mechanism for seatbelts (*ELEMENT_SEATBELT_SLIPRING) with belt material (*MAT_SEATBELT) to standard truss beam or cable elements (*ELEMENT_BEAM with ELFORM=3 or 6) with muscle material (*MAT_MUSCLE) and cable material (*MAT_CABLE_DISCRETE):

  → New keyword ***ELEMENT_BEAM_PULLEY**

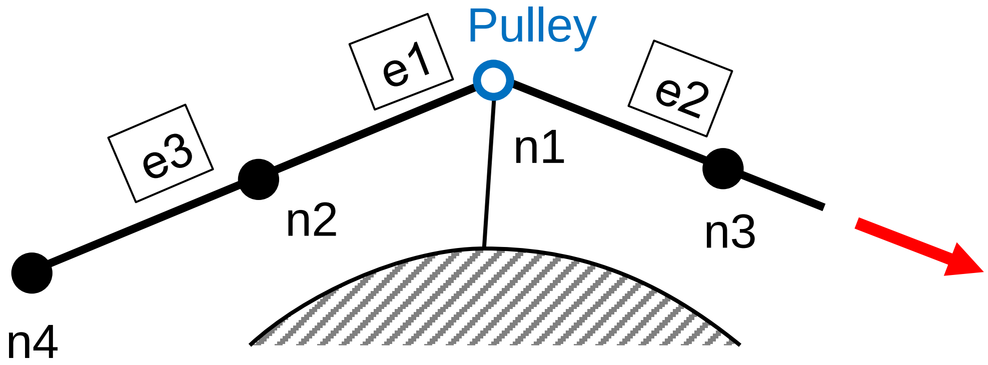
- **Definition:** Pulleys allow continuous sliding of a string of truss beam elements through a sharp change of angle. To define a pulley, two beam elements which meet at the pulley, a friction coefficient $\mu$, and a pulley node have to be identified. The two elements must have a common node coincident with the pulley node.

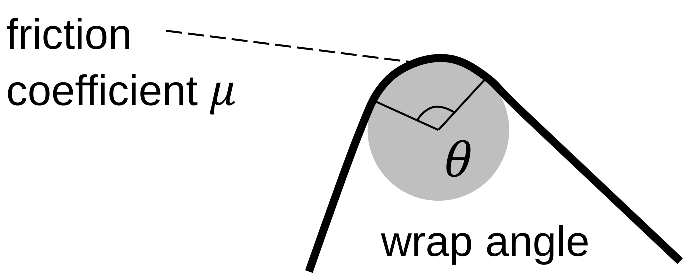
# *ELEMENT_BEAM_PULLEY

| Card | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | PUID | BID1 | BID2 | PNID | FD | FS | LMIN | DC |
| Type | I | I | I | I | F | F | F | F |
| Default | 0 | 0 | 0 | 0 | 0.0 | 0.0 | 0.0 | 0.0 |

| | |
|---|---|
| PUID | Pulley ID. |
| BID1 | Truss beam element 1 ID. |
| BID2 | Truss beam element 2 ID. |
| PNID | Pulley node, NID. |
| FD | Coulomb dynamic friction coefficient. |
| FS | Optional Coulomb static friction coefficient. |
| LMIN | Minimum length. |
| DC | Decay constant. $\mu_c = FD + (FS - FD)e^{-DC \cdot \lvert v_{rel} \rvert}$ |

Beam material (in the form of unstretched length) is passed from element e1 to element e2 to achieve **slip**.

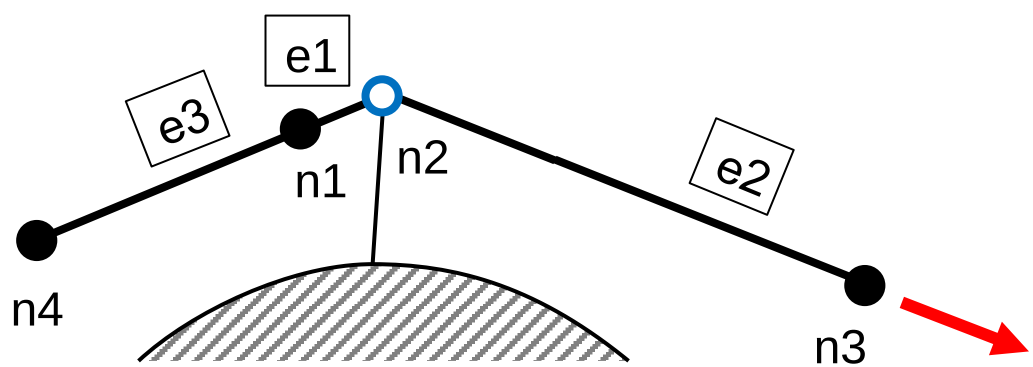


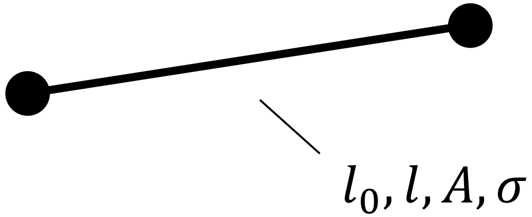

slip condition: $T_2 \leq T_1 e^{\mu\theta}$

*„capstan equation"*

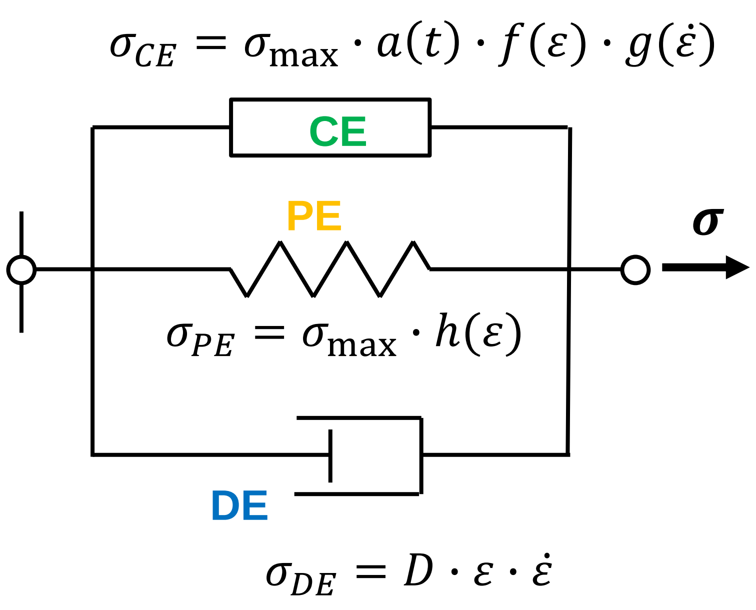
If unstretched length of e1 < $l_{\min}$, the beam gets remeshed locally: short element passes through pulley and reappears on the other side: **"swap"**

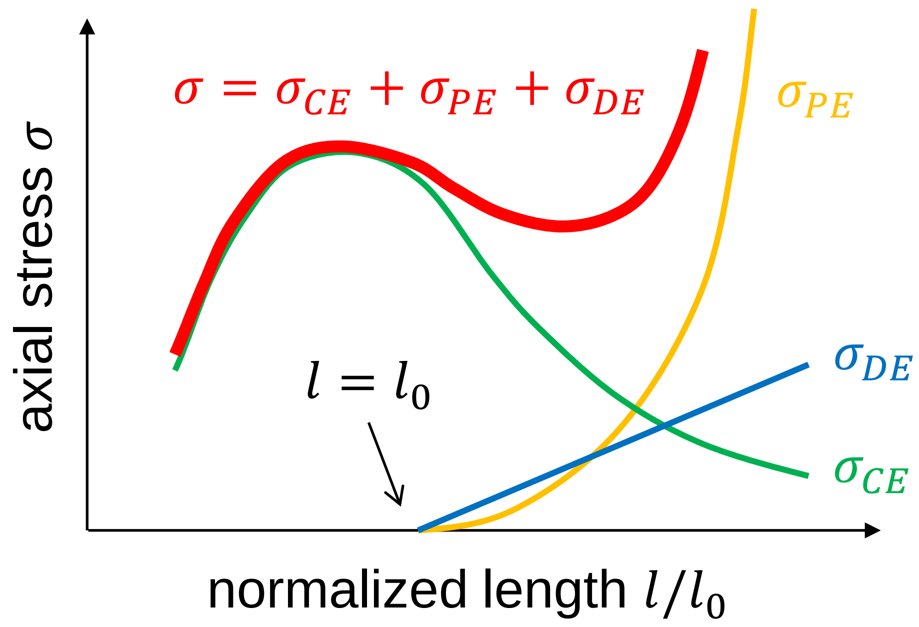


- **Truss beam or cable element:**
  - pin-jointed element with 3 degrees of freedom at each node
  - axial force depends on $l_0$, $l$, $A$, and the constitutive model
  - 6 material models for the truss element: elastic (*MAT_001), elastic-plastic (*MAT_003), elastic-plastic thermal (*MAT_004), Mooney-Rivlin rubber (*MAT_027), simplified Johnson-Cook (*MAT_098), and Hill's muscle model (*MAT_156).
  - For the cable element, a nonlinear elastic model (*MAT_071) exists.
  - For the beam pulley, materials **1**, **71**, and **156** are implemented at the moment. Other materials can be added in a modular way in the future.



- **Muscle material:** This material is a Hill-type muscle model: *MAT_156. The discrete (rheological) model is a parallel arrangement of a contractile element (CE), a passive element (PE) and a damper element (DE).
  - **contractile element**: force generation by the muscle
  - **passive element**: energy storage from muscle elasticity
  - **damper element**: muscular viscosity



- **Pulley algorithm:** Overall flow diagram.

Loop over all truss beam elements; for each:
- compute deformation $l/l_0$ and strain rate $\dot{\varepsilon}$,
- determine axial stress: $\sigma = \hat{\sigma}(l, l_0, \dot{\varepsilon}, \ldots)$,
- calculate axial force $T = A\sigma$

↓

Loop over all pulleys; for each:
- check slip condition $T_2 \leq T_1 e^{\mu\theta}$
- if condition is not met, compute correct slip (nonlinear iteration procedure)
- calculate new axial forces with correct slip
- if unstretched length reaches $l_{\min}$, swap element from one side to the other

1. **Standard force computation for each truss beam element**

- Get unstretched length $l_0$
- Compute current length $l$ and strain rate $\dot{\varepsilon}$
- Calculate axial stress as a function of $l_0, l, \dot{\varepsilon},$ and material parameters:

$$\sigma = \sigma(l, l_0, \dot{\varepsilon}, \ldots) = \sigma_{CE} + \sigma_{PE} + \sigma_{DE}$$

- Compute axial force $T = T(l, l_0, \dot{\varepsilon}, \ldots) = A\,\sigma(l, l_0, \dot{\varepsilon}, \ldots)$

+ Store relevant beam data (lengths, stress, strain rate, ...) for possible later use in pulley computation

2. **Force and length correction for each truss pair adjacent to a pulley**

- Use computed forces as trial values: $T_1^{trial}$, $T_2^{trial}$
- Check slip condition: $T_2^{trial} \leq T_1^{trial} e^{\mu\theta}$
- If slip condition is met: $T_1 = T_1^{trial}$, $T_2 = T_2^{trial}$, done.
- If slip condition is not met, use Brent's method to find root of non-linear slip function, i.e. solve for unknown amount of slip $\Delta l$:

$$\frac{T_2(l, l_0 + \Delta l, \dot{\varepsilon}, \ldots)}{T_1(l, l_0 - \Delta l, \dot{\varepsilon}, \ldots)\, e^{\mu\theta}} - 1 = 0$$

- During this iteration, the muscle material model is called twice (two elements) in each iteration step
- Update unstretched lengths of elements e1 ($l_0 - \Delta l$) and e2 ($l_0 + \Delta l$)
- Use corrected axial forces $T_1$ and $T_2$ and store history

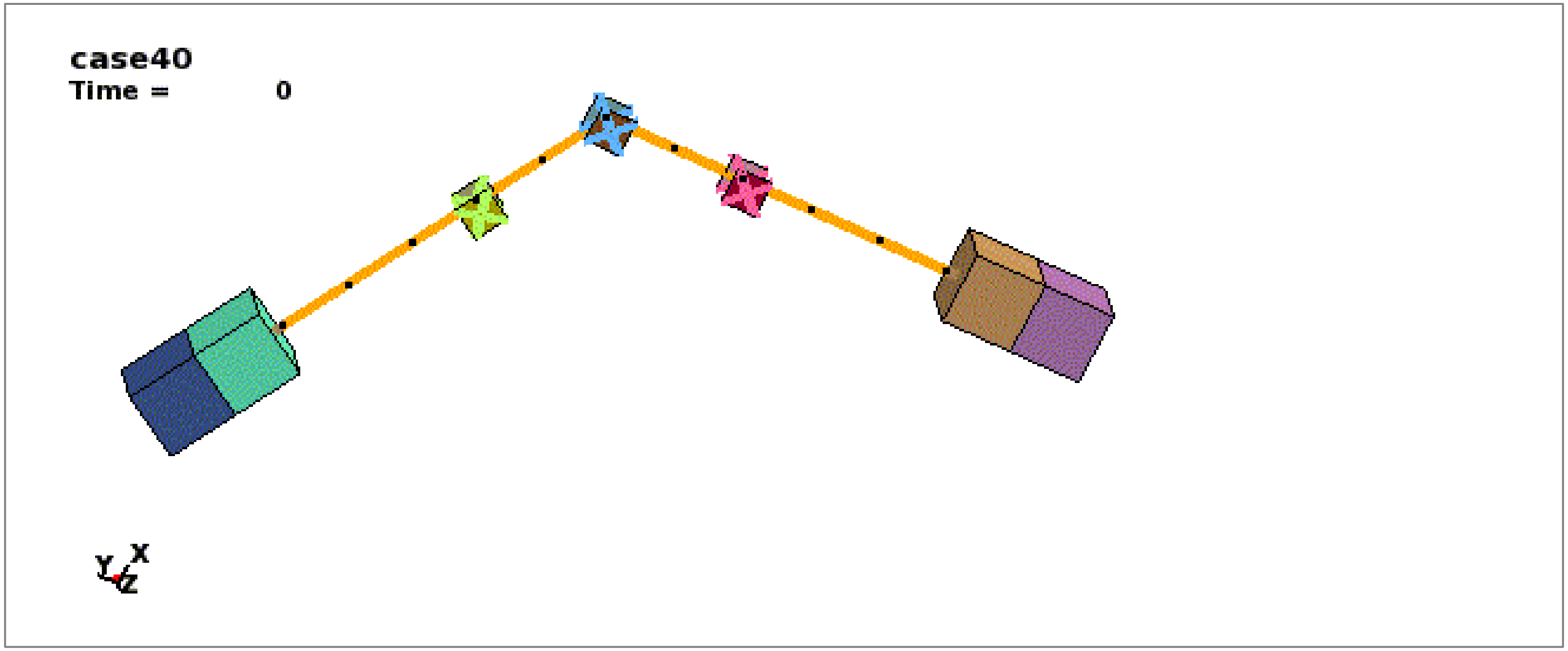
3. **Swap short element from one side to the other**

- If unstretched length $l_0 < l_{\min}$ , swap element
- Therefore, change connectivity as shown before.
- Pulley node becomes n2, and node n1 moves to new location:

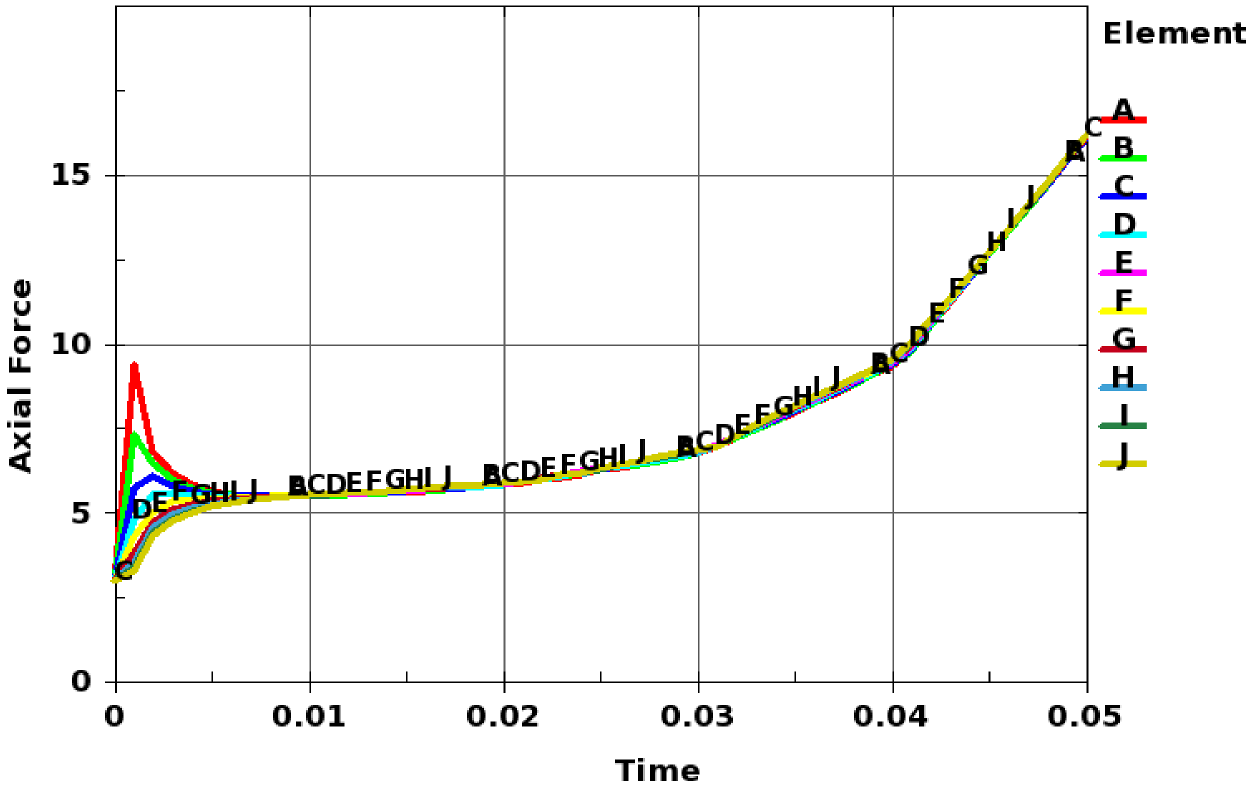
$$\mathbf{x}_{n1} = \mathbf{x}_{n2} + 1.1\, l_{\min}\, \mathbf{n}_{e2}$$

- Update velocity of the new node n1 depending on slip and on velocities of nodes n2 and n3.
- Modify element properties for moved element, changing force and history variables to be the same as the element on the side to which the element has moved.
- Force and strain in elements e2 and e3 are unchanged.

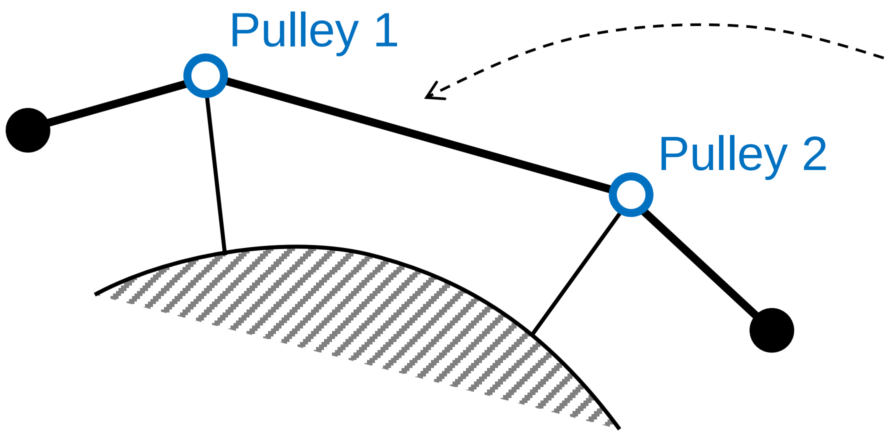
- With new keyword ***ELEMENT_BEAM_PULLEY**, smooth results can be achieved (no contact used):



- With new keyword ***ELEMENT_BEAM_PULLEY**, uniform axial forces can be achieved:



- **Remark 1:** Situations without a node between two pulleys should be avoided




*element in between gets different informations from each pulley; this can lead to problems; finer mesh is needed*

- **Remark 2:** Pulley element available since R6, upcoming release R7 will contain some bug fixes for *MAT_MUSCLE with SVR<0 (curve for stress vs. strain rate).

- **Remark 3:** ASCII result file **pllyout** (*DATABASE_PLLYOUT) contains slip length, slip rate, resultant force, and wrap angle.

## Summary

- Computational method for continuous sliding of rope-type structures
- Developed for biomechnical applications (muscle strands or tendons)
- But also applicable for all kinds of pulley-like mechanisms
- Integration of material model in modular fashion
- Straightforward extension to other material laws