Updated frequency domain analysis in LS-DYNA®

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1 Overview

A series of frequency domain features have been implemented to LS-DYNA since version 971 R6. These features can be used to perform vibration, acoustic and fatigue analysis for users from various industries [1]. These features include

- FRF (frequency response functions)
  Keyword: ***FREQUENCY_DOMAIN_FRF**
- SSD (steady state dynamics)
  Keyword: ***FREQUENCY_DOMAIN_SSD**
- Random vibration (fatigue)
  Keyword: ***FREQUENCY_DOMAIN_RANDOM_VIBRATION_*{FATIGUE}***
- Response spectrum analysis
  Keyword: ***FREQUENCY_DOMAIN_RESPONSE_SPECTRUM**
- BEM acoustics
  Keyword: ***FREQUENCY_DOMAIN_ACOUSTIC_BEM**
- FEM acoustics
  Keyword: ***FREQUENCY_DOMAIN_ACOUSTIC_FEM**

These features can find application in

- NVH of automotives and air planes
- Acoustic design and analysis of buildings and products
- Defense industry
- Fatigue of machines and engines
- Safety evaluation of civil and hydraulic structures
- Earthquake engineering
- Offshore industries
- Many others

Regarding post-processing of the frequency domain analysis, a series of ASCII and BINARY databases have been implemented.

The ASCII databases include

- FRF_AMPLITUDE (for FRF analysis)
- FRF_ANGLE (for FRF analysis)
- Press_Pa (for acoustic analysis)
- Press_dB (for acoustic analysis)
- NODOUT_PSD (for random vibration)
- NODOUT_SSD (for steady state dynamics)
- NODOUT_SPCM (for response spectrum analysis)
- ELOUT_PSD (for random vibration)
- ELOUT_SSD (for steady state dynamics)
- ELOUT_SPCM (for response spectrum analysis)

Particularly the NODOUT (_PSD, _SSD, _SPCM) and ELOUT (_PSD, _SSD, _SPCM) databases are dumped into binary file BINOUT and one can use LS-PrePost or l2a.exe to extract them from BINOUT.

The Nodes whose nodal results are output to NODOUT_ databases are specified by card ***DATABASE_HISTORY_NODE**. The solid, beam, shell and thick shell elements whose elemental results are output to ELOUT_ databases are specified by the following cards:

DATABASE_HISTORY_SOLID_*{OPTION}
DATABASE_HISTORY_BEAM_*{OPTION}
DATABASE_HISTORY_SHELL_*{OPTION}
DATABASE_HISTORY_TSHELL_*{OPTION}

For BINARY plot databases, they are activated by keyword
DATABASE_FREQUENCY_BINARY_*{OPTION}, where the ***{OPTION}*** can be any one of the following databases:

Database	Lspcode	Used for
D3SSD	21	Steady state dynamics
D3SPCM	22	Response spectrum analysis
D3PSD	23	Random vibration PSD
D3RMS	24	Random vibration RMS
D3FTG	25	Random vibration fatigue
D3ACS	26	FEM acoustics
D3ATV	27	BEM acoustic transfer vector

Table 1: New binary databases for frequency domain analysis.

These databases are output in the same format as D3PLOT, and are accessible to LS-PrePost. Particularly the parameter lspcode (saved in the header file of the database) is a flag to tell LS-PrePost or other post-processing softwares that what kind of database it is. As can be seen from Table 1, the content of these databases is different from those in D3PLOT. Thus some updates or revision from LS-PrePost or other post-processing softwares are needed to get them working appropriately with these new databases. The updates or revision include marking the x-axis of animation as "frequency" instead of "time", and correcting the name of the variables … Some of such updates or revisions have been accomplished in LS-PrePost.

Some updates have been made to these frequency domain analysis features since the last forum. These updates were made to extend the capabilities of frequency domain analysis of LS-DYNA, or to improve the computational performance. A brief introduction of the updates, accompanied by several examples, is provided in the following sections of the paper.

2 ATV and MATV techniques for BEM Acoustic solvers

A bunch of BEM Acoustic solvers (collocation BEM, variational indirect BEM, dual BEM with Burton-Miller formulation, Rayleigh method and Kirchhoff method) have been implemented to LS-DYNA [2]. They are used to predict the radiated noise from a vibrating structure.

To facilitate the acoustic analysis for structures which are subjected to multiple loading cases, two new techniques ATV (Acoustic Transfer Vector) and MATV (Modal Acoustic Transfer Vector) have been implemented to the BEM acoustic solvers.

2.1 ATV

ATV is defined as the transfer function between the normal nodal (or elemental) velocity and the acoustic pressure at field points. For example, the acoustic pressure at field point *i* due to unit normal velocity at node *j* on the structure surface can be expressed as $\Omega_{i,j}$. This provides Acoustic Transfer Vector from structural node *j* to field point *i* in acoustic volume. One should note that ATV is a function of frequency. It only depends on the properties of acoustic medium (density, sound speed), geometry of structures, and location of field points. It is not dependent on the real loading condition.
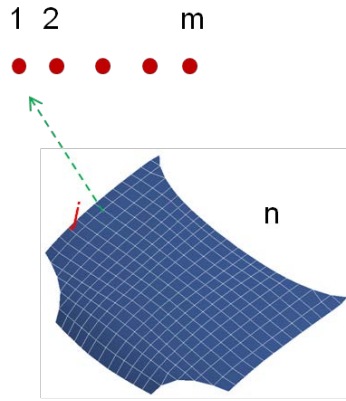
Fig. 1: Structural surface nodes and field points for acoustic computation.

For the structure shown in Figure 1, if there are n nodes on the surface and m field points in acoustic volume, the ATV matrix can be expressed as

$$[ATV]_{m \times n} = \begin{bmatrix} \Omega_{1,1} & \Omega_{1,2} & \cdots & \Omega_{1,j} & \cdots & \Omega_{1,n} \\ \Omega_{2,1} & \Omega_{2,2} & \cdots & \Omega_{2,j} & \cdots & \Omega_{2,n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \Omega_{i,1} & \Omega_{i,2} & \cdots & \Omega_{i,j} & \cdots & \Omega_{i,n} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \Omega_{m,1} & \Omega_{m,2} & \cdots & \Omega_{m,j} & \cdots & \Omega_{m,n} \end{bmatrix} \tag{1}$$

Once the ATV matrix is obtained, for given vibration condition on structure surface, the acoustic pressure at the m field points can be computed by simple matrix-vector multiplication, as follows

$$\{P\}_m = [ATV]_{m \times n} \{v\}_n \tag{2}$$

Where $\{P\}_m$ is the acoustic pressure vector at the m field points; $\{v\}_n$ is the normal velocity vector at the n surface nodes (or elements);

The ATV can be plotted on the structural surface for visualization. A new database D3ATV has been implemented in LS-DYNA. The database plots the real part, the imaginary part and the SPL (Sound Pressure Level, in unit of dB) of ATV, for each field point and each frequency. Figure 2 shows real part of ATV for a simplified engine model for the field point 284900 and the frequency 100 Hz.
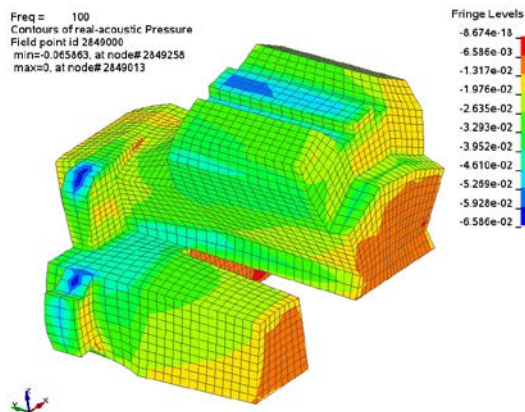

Fig. 2: D3ATV for a simplified engine model.

2.2 MATV

If the dynamic response of structures can be obtained using the modal superposition method (for example, using keyword ***FREQUENCY_DOMAIN_SSD** in LS-DYNA), the normal velocity vector $\{v\}_n$

in (2) can be obtained as a product of round frequency ω, modal shape matrix and modal coordinates vector. Accordingly the equation (2) can be revised as

$$
\begin{aligned}
\{P\}_m &= [ATV]_{m \times n} \{v\}_n \\
&= [ATV]_{m \times n} i\omega \{u\}_n \\
&= [ATV]_{m \times n} i\omega [\phi]^T_{n \times l} \{q\}_l \\
&= [MATV]_{m \times l} \{q\}_l
\end{aligned}
\tag{3}
$$

Where i is the imaginary unit ($i = \sqrt{-1}$) and $\{u\}_n$ is the displacement vector. The matrix $[\phi]_{n \times l}$ is the modal shape matrix, provided by implicit modal analysis. $\{q\}_l$ is the modal coordinates (assuming l modes are used); $[MATV]_{m \times l}$ is the MATV matrix, which is constant for each given frequency. For each excitation frequency f, LS-DYNA will generate the psedo-velocity boundary condition $i\omega \{\phi\}_j \ (\omega = 2\pi f), \ j = 1, \ldots, l$ and run BEM acoustic computation for the m field points to get the MATV matrix.

For each load case, only the modal coordinates vector $\{q\}_l$ need to be updated. Once it is ready, a simple matrix-vector multiplication can provide solution for acoustic pressure at the m field points. For a real problem, the number of eigen modes involved in modal superposition is usually much less than the number of nodes (or elements) in the boundary elements ($l << n$). So the MATV approach represented by equation (3) is more efficient than the ATV approach represented by equation (2), if the vibration simulation can be accomplished by modal superposition. This is due to the fact that the effort to get MATV matrix is much less than the effort to get ATV matrix.

There are 2 steps involved in using the MATV technique in BEM Acoustics:

Step 1: generating MATV matrices.

The keywords ***CONTROL_IMPLICIT_GENERAL**, ***CONTROL_IMPLICIT_EIGENVALUE** are used to perform implicit modal analysis, to get eigen modes $\{\phi\}_j$. Then the keyword ***FREQUENCY_DOMAIN_ACOUSTIC_BEM_MATV** is used without specifying any boundary condition, to run acoustic computation for each psedo-velocity boundary condition $i\omega \{\phi\}_j$ for each excitation frequency. The MATV matrices are saved in binary scratch file "bin_bepressure".

Step 2: acoustic computation for each load case.

The keywords ***FREQUENCY_DOMAIN_SSD**, ***FREQUENCY_DOMAIN_ACOUSTIC_BEM_MATV** are used, both with a restart option. For SSD, it restarts with existing d3eigv from step 1 (restmd=1); for MATV BEM, it restarts with existing MATV matrices, saved in "bin_bepressure", given by step 1 (restrt=1). Moreover, one can define multiple load cases in one input deck, by taking advantage of the convenient CASE scheme. For example, the first load case (defined by ***FREQUENCY_DOMAIN_SSD** with corresponding load condition) can be put in the section between ***CASE_BEGIN_1** and ***CASE_END_1**. For the other load cases, they can be similarly put in the CASE section one by one. See Figure 3 for an example.

```
*FREQUENCY_DOMAIN_ACOUSTIC_BEM_MATV
$#        ro           c       fnmin      fnmax       nfreq     dt_out    t_start       pref
    1.210000  340.00000    1.000000   500.0000         500   0.001000      0.000  2.0000E-5
$#nsid_ext    type_ext    nsid_int   type_int     fft_win      trslt     ipfile     iunits
        3           1           0          0           4          0          0          0
$#  method       maxit    tol_iter        ndd      tol_lr   tol_fact      ibdim        npg
        2        1000  1.0000E-6          8       0.000      0.000          0          0
$#                 nbc     restrt       iedge       noel      nfrup
                    1          1
$#    ssid      sstype        norm   bem_type    restart      iedge       noel      nfrup
        34           2
*CASE_BEGIN_1
*FREQUENCY_DOMAIN_SSD
$#   mdmin       mdmax       fnmin      fnmax      restmd     restdp
        1          15       0.000      0.000          1          0
$#   dampf       lcdam       lctyp     dmpmas     dmpstf
  0.020000          0           0      0.000      0.000
$#                                                             nout      notyp       nova
                                                                34          2          0
$#      pid        ntyp         dof        vad        lc1        lc2     lcflag        vid
      123           1           2          0        100        200          0          0
*DEFINE_CURVE
$      LCID        SIDR      SCALEX     SCALEY       OFFA       OFFO     DATTYP
       100           0         1.0        1.0        0.0        0.0          0
$                    X                      Y
      1.00000000000          146.766998291
      ...
       100.000000000          513.684020996
*CASE_END_1
*CASE_BEGIN_2
*FREQUENCY_DOMAIN_SSD
$#   mdmin       mdmax       fnmin      fnmax      restmd     restdp
        1          15       0.000      0.000          1          0
$#   dampf       lcdam       lctyp     dmpmas     dmpstf
  0.020000          0           0      0.000      0.000
$#                                                             nout      notyp       nova
                                                                34          2          0
$#      pid        ntyp         dof        vad        lc1        lc2     lcflag        vid
      124           1           2          0        101        201          0          0
*DEFINE_CURVE
$      LCID        SIDR      SCALEX     SCALEY       OFFA       OFFO     DATTYP
       101           0         1.0        1.0        0.0        0.0          0
$                    X                      Y
      1.00000000000           96.042098999
      ...
       100.000000000          2602.51000977
*CASE_END_2
```

Fig. 3: Defining multiple load cases with CASE for running MATV BEM.

Then with a single LS-DYNA run (with flag CASE in the command line), one can get the solution of acoustic pressure and SPL for all the load cases (e.g. case1.Press_Pa, case1.Press_dB, case2.Press_Pa, case2.Press_dB, ...)
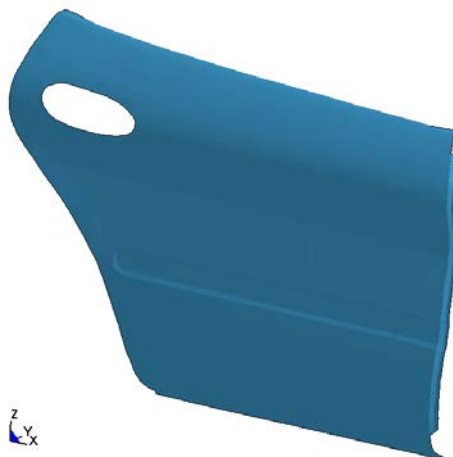

Fig. 4: using MATV for a door model.

For a simplified door model shown in Figure 4, benchmark testings are performed to check the accuracy and efficiency of the MATV method, comparing with the traditional BEM.

Figure 5 shows the SPL (dB) value of the noise at a field point, for the case with a harmonic nodal force excitation applied on the door. The excitation is given in the frequency range of 100-500 Hz, with 101 equally spaced frequencies. For running implicit modal analysis, normal modes up to 600 Hz is

used, which is 20% higher than the maximum excitation frequency. The solution of the problem is based on the combination of SSD and BEM acoustics. Two sets of results are given in Figure 5: One is obtained with MATV BEM, and the other is obtained with traditional BEM. The two sets of results are actually identical, as illustrated by the Figure.
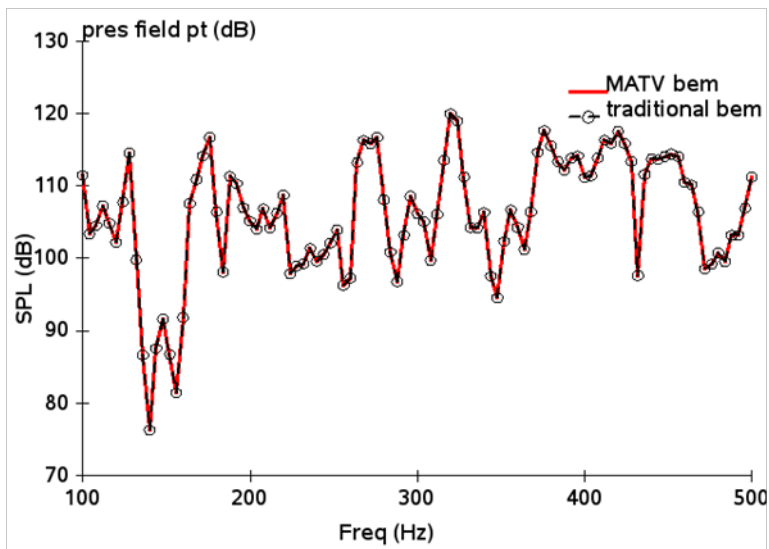

Fig. 5: SPL at field point.

To study the efficiency of the MATV BEM, we consider 1 load case and 10 load cases, by three approaches. The computation is performed on Intel Xeron CPU E5504 @2.00 GHz (CPU MHz: 1596.00 cache size 4096 KB). The three approaches are: 1) SSD + traditional BEM, which means that LS-DYNA goes through the whole procedure (modal analysis, SSD and traditional BEM) for each load case; 2) Restart SSD + traditional BEM, which means that starting from the 2nd load case, the modal analysis is skipped (since the D3EIGV binary database has been generated during the solution for the first load case) and LS-DYNA runs a restart SSD and then traditional BEM; 3) Restart SSD + MATV BEM, which means that LS-DYNA skips the modal analysis part starting from the 2nd load case, and runs a restart SSD and then uses the MATV based BEM to get the acoustic pressure for all the load cases.

As shown in Table 2, the approach 3) Restart SSD + MATV BEM shows significant saving in CPU cost comparing with the other two approaches, when 10 load cases are considered. When there is only 1 load case, the MATV BEM is slower than the other two approaches, since the computation and the saving of MATV matrices take some extra CPU time. However, once the MATV matrices are ready the solution for the additional load cases takes only a tiny CPU time.

Cases	1) SSD + traditional BEM	2) Restart SSD + traditional BEM	3) Restart SSD + MATV BEM
1 load case	2 h 39 m 50 s	2 h 39 m 50 s	4 h 40 m 56 s
10 load cases	26 h 38 m 18 s	25 h 53 m 13 s	4 h 41 m 10 s

Table 2: CPU time for acoustic computation of door model.

3 Incident acoustic wave

A new keyword ***FREQUENCY_DOMAIN_ACOUSTIC_INCIDENT_WAVE** has been introduced to consider incident acoustic waves. The incident wave is useful in modeling sonar system in a submarine and explosive waves.

The keyword format is

Card 1	1	2	3	4	5	6	7	8
Variable	TYPE	MAG	XC	YC	ZC			
Type	I	F	F	F	F			
Default	1	none	none	none	none			

For plane wave (TYPE=1), the incident wave is defined by

$$p^i = Ae^{-ik(\alpha x + \beta y + \gamma z)} \tag{4}$$

For spherical wave (TYPE=2), the incident wave is defined by

$$p^i = A\frac{e^{-ikR}}{R} \tag{5}$$

In equations (4) and (5), A is magnitude or strength of incident wave (parameter MAG in the keyword); k is wave number (= ω/c, where ω is round frequency and c is wave speed); α, β and γ are directional cosines for plane wave (see equation (4)), and are defined by (XC, YC, ZC) from the keyword. R is distance between spherical source and field point for spherical incident wave; (XC, YC, ZC) define the center of the spherical wave or the source point.

One can define multiple incident waves in one model, by repeating the keyword ***FREQUENCY_DOMAIN_ACOUSTIC_INCIDENT_WAVE**, or simply repeating the Card 1 in the keyword.

A benchmark example of sound scattering on rigid sphere is adopted to validate the implementation. See Figure 6 below.

Fig. 6: Sound scattering on rigid sphere.

The spherical source is located at 4r from the center of the rigid sphere (r is the radius of the rigid sphere). Two field points are located at 5r from the center of the rigid sphere. One is on the same side of the source (Field point A), and the other is located on the opposite side (Field point B). We compare the real part and imaginary part of the pressure, given by LS-DYNA and by analytical solution (given as infinite series expansion, truncated in computation) [3].

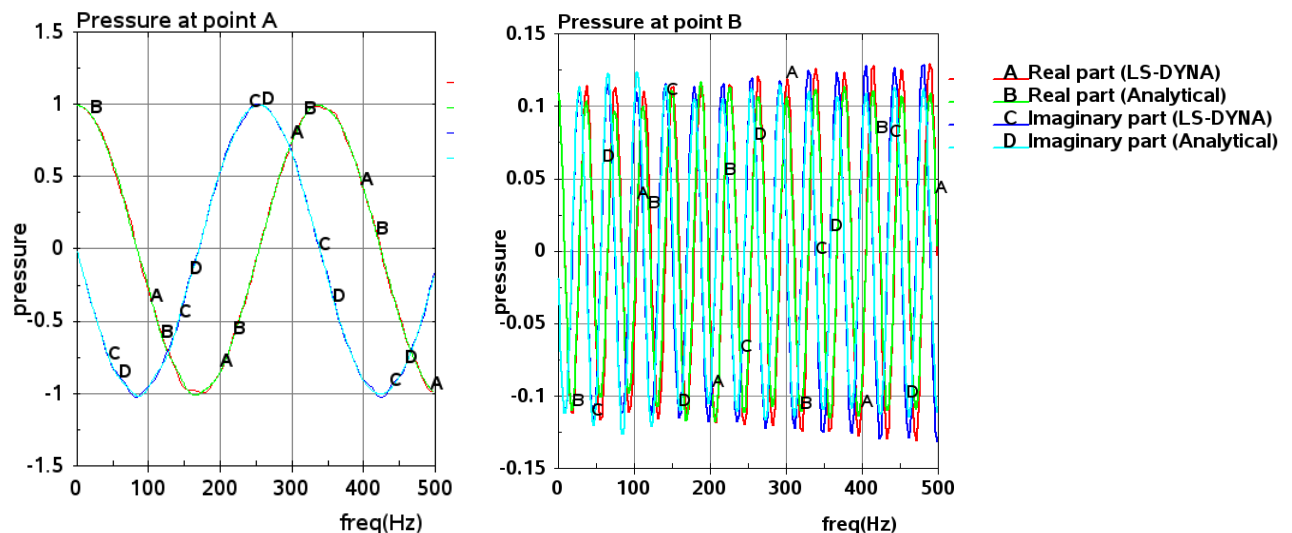LS-DYNA results match very well with the analytical solution, as shown in Figure 7.

Fig. 7: Acoustic pressure at the two field points.

4 Frequency dependent complex sound speed

To take into account damping in the acoustic system, a new keyword ***FREQUENCY_DOMAIN_ACOUSTIC_SOUND_SPEED** is introduced to LS-DYNA, to allow defining the frequency dependent complex sound speed by two load curves.

$$c(f) = c_r(f) + ic_i(f) \tag{6}$$

For a simple muffler model shown in Figure 8, a unit normal velocity boundary condition is prescribed at one end. An impedance boundary condition is given at the other end. The rest of the muffler surface is assumed to be rigid. The range of frequency under study is 10-1000 Hz. The acoustic pressure at one field point inside the muffler is computed by LS-DYNA. One can see that the peak dB values of the acoustic pressure are reduced by using complex sound speed.
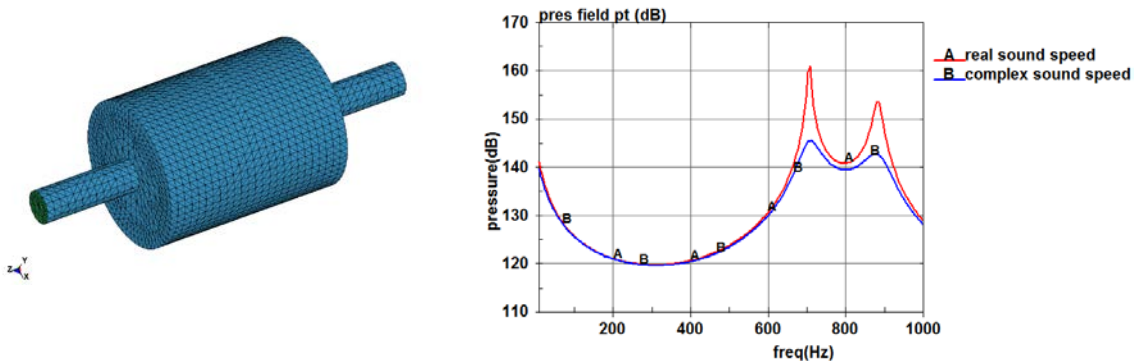

Fig. 8: Acoustic pressure due to real and complex sound speed.

5 Fatigue analysis based on SSD (Steady state dynamic analysis)

Fatigue is the weakening of material due to repeated or cyclic loading. Fatigue failure under harmonic or steady state vibration condition is very common in various industries, e.g. in a sine sweep test.
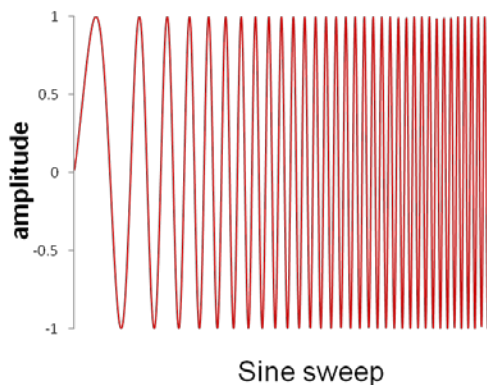

Fig. 9: A sample sine sweep test load curve.

A fatigue analysis method is implemented based on steady state vibration condition.

The corresponding keyword is ***FREQUENCY_DOMAIN_SSD_{FATIGUE}**.

This feature is based on the rainflow counting algorithm and the material's S-N fatigue curve. The rainflow counting algorithm is used to get the number of stress cycles for Von-Mises stress for each excitation frequency. Linear superposition is employed to get the total fatigue damage under different stress levels.

The cumulative damage ratio *R* can be expressed as

$$R = \sum_i R_i = \sum_i \frac{n_i}{N_i} \tag{7}$$

Where R_i is the damage ratio due to stress level *i*, n_i is the actural number of cycles for stress level *i*,

and N_i is the number of cycles for fatigue failure for stress level *i* (obtained from material's S-N

curve). *R* is a real number larger than 0. If *R* is equal to or larger than 1, it means that the material has failed due to fatigue.

In high-cycle fatigue situation, the material's fatigue behavior is usually characterized by an S-N curve, which is also known as a Wöhler curve. To define the material's S-N curve, a new keyword

*MAT_ADD_FATIGUE is added to LS-DYNA. Three options are available for defining the S-N fatigue curve.

1) By curve ID (see ***DEFINE_CURVE**)
2) By equation $N \cdot S^m = a$
3) By equation $\log(S) = a - b \cdot \log(N)$

When the S-N fatigue curve is defined by options 2) or 3), the parameters *a* and *m* (in option 2) or *a* and *b* (in option 3) are constants which are dependent on material model.

For a model shown in Figure 10, the cumulative damage ratio is computed and plotted in D3FTG, which is accessible to LS-PrePost. The loading condition is given as base acceleration spectrum, see Table 3.

To get the solution for SSD, implicit modal analysis is first performed for the structure. The first 300 Normal modes, which provides natural frequency up to 2703Hz (35% higher than the maximum frequency 2000 Hz for excitation) are used in SSD.

Frequency (Hz)	Acceleration (g)	Duration (minute)
16	0.5	10
20	0.5	10
25	0.5	10
31.5	0.5	10
...	...	...
2000	0.5	10

Table 3: Loading condition.

The material's S-N fatigue curve is defined as

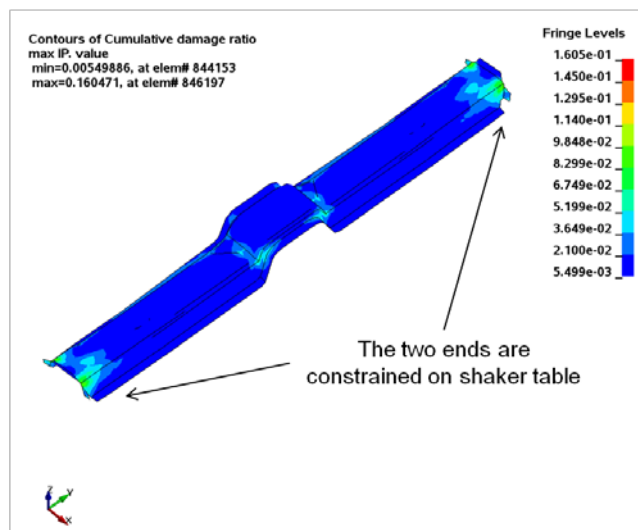| σ (MPa)	N
100	8×10^4
10	8×10^5
1	8×10^6
0.1	8×10^7
0.01	8×10^8

Table 4: S-N fatigue curve.

Fig. 10: Cumulative damage ratio for the beam under SSD.

As shown in Figure 10, the two ends of the structure, which are constrained to the shaker table, are characterized with higher cumulative damage ratios, which suggests a more severe damage to the material. But the peak value of the cumulative damage ratio is still less than 1. It means that the structure is still safe after the whole loading process.

6 Summary

A list of updated frequency domain features in LS-DYNA are reviewed in the paper. They include ATV and MATV techniques for BEM Acoustic solvers; incident waves in acoustic analysis; using frequency dependent complex sound speed in acoustic analysis and fatigue analysis based on SSD (steady state dynamics). Several examples are provided to demonstrate the effectiveness of the updated features.

7 Literature

[1]   LS-DYNA Keyword User's Manual, Livermore Software Technology Corporation, 2014.

[2]   Yun Huang, Mhamed Souli, Rongfeng Liu, BEM Methods for acoustic and vibroacoustic problems in LS-DYNA. Proceedings of the 11th International LS-DYNA Users' Conference, Simulation (2), 4-1 – 4-8, 2010.

[3]   Yueping Guo, Computation of Sound Propagation by Boundary Element Method. NASA Contract Report, NAS1-00086-A003, 2005.